MATH

ASSESSMENTS:

1. Informally assess the student's understanding of the Piagetian concepts that are critical foundations of mathematical concepts and relations: object permanence, conservation of matter, reversibility, seriation, one-to-one correspondence, and classification.

2. Evaluate the student's understanding of the concepts underlying addition and subtraction in a variety of situations and with a variety of materials. Make sure she understands that numbers represent the quantity of any type of entity within a set, that addition is the combining of sets, and that zero also represents a set that has nothing in it (the null set). Before the student will be able to make sense of the value of money and equivalencies in money (e.g., the number of nickels in a quarter and the reason) and measurement (e.g., time: seconds, minutes, hours, days, weeks, months, years), she must understand sets and how they can be combined.

3. Before teaching math facts, give the student a timed test to see what facts she can complete correctly within 2 minutes, an oral test using flashcards to see which facts she can answer within 3 seconds. Use the results to develop a program for fact learning. Have the student chart her progress as she masters new facts

4. To assess the student's conceptual understanding of basic computation, give her a worksheet with a few addition, subtraction, multiplication, and division problems. Have her work out the problems using concrete objects, such as beans or marbles. Ask her to think out loud as she sets up and works the problems

5. When it is unclear why the student is missing specific computational problems, conduct an oral interview with her. Ask her to talk through the steps as she solves the problems.

6. When the student makes errors on word problems or computation, analyze the errors to find the component skills or prerequisite information she is missing and/or the rules she misunderstands. Without intervention, the student will continue to make the same systematic errors.

7. Use task analysis to help the student master computational algorithms. Identify the algorithm to be learned. List and arrange all the prerequisite skills and the steps needed to perform the algorithm into a logical teaching sequence. Determine exactly which steps the student cannot perform through informal testing and then teach the steps in sequence.

8. Assess the student's comprehension of algebra word problems by providing problems accompanied by tables representing the variables involved in each problem. Ask the student to fill in the values of the variables based on the information given in the problem. The student's ability to associate the variables and values indicates her comprehension of the problem.

INTERVENTION IDEAS:

1. The student will explore new concepts with concrete materials prior teaching concepts.
*Teach all new concepts and extensions of known concepts first with concrete materials, making sure that the student has a chance to experiment with the materials and understands the new concept before moving to the next level of abstractness. For example, when teaching simple fractions, use a variety of materials (e.g., tiles, pizza, Cuisenaire rods) to represent the whole and the fraction. Make sure she understands or learns the related terminology (e.g., numerator, denominator) by having her talk about what she is doing with the materials. Provide plenty of practice in manipulating the materials to solve problems. Then present similar types of problems using pictures and requiring her to draw pictures to represent the problems posed. When she has mastered this step, associate these materials with numbers. If necessary, tallies may be used as a level between pictures and symbols. The last step is to use numbers alone to represent new concepts. [Adapted from Principles and Standards for School Mathematics, National Council of Teachers of Mathematics, 2000]

2. The teacher will utilize the Guided Discovery Approach in addition to the core program to increase the student's mathematical skills.
*If the student appears to have a strength in inductive reasoning, use a guided discovery approach to teaching new concepts and extensions of new concepts. Guide the student to solve problems and then to generate a rule based on her observations. Then have her test the rule in further examples.

3. The teacher will utilize a Rule-based Approach in addition to the core program to increase the student's mathematical skills.
*If the student appears to be stronger in deductive reasoning than in inductive reasoning, use a rule-based approach in teaching new concepts and extensions of new concepts. Explain the concept/extension to the student, demonstrate how it works, state the rule governing it, ask questions to assess understanding, and have the student practice its application.

4. The student will recognize mathematical relationships beginning with verbal scripting by the teacher and moving to student articulation of conceptual understanding.
*If the student has difficulty understanding procedures in math activities and does not intuit mathematical relationships. Spell them out explicitly and repeatedly, gradually asking her what she is to do next until she can verbalize the procedure. Then have her practice the procedure until it is memorized.

5. The student will observe and practice self-talk strategies.
*When introducing new concepts and skills, use modeling and demonstrations. Have the student watch you perform the task as you talk yourself through it and then have her perform the task as you talk it through.

6. The student will demonstrate mastery of skills in isolation through additional practice.
*Supplement the student's basal math textbook with additional examples and practice exercises. Allow the student to move on in the text only when she has demonstrated mastery of the current skills.

7. The student will maintain skill development through scheduled review and guided practice.
*The student requires frequent review and reinforcement of concepts and procedures learned. Begin each lesson with a review of the mathematical skills and concepts covered the previous day and, additionally, provide weekly and monthly reviews.

TERMINOLOGY:

1. The student will use quantitative terms to describe classification rules.
*While teaching classification skills, use each attribute that the student identifies as a springboard for teaching quantitative terms such as *big*, *small*, *narrow*, *wide*, *tall*, *short*, *thick*, and *thin*. Also teach the terms for comparative concepts such as *big*, *bigger*, and *biggest*.

2. The student will recognize key terms and their meaning.
*Teach the student that several terms can indicate the same process. For example, *adding*, *plus*, and *all* can refer to the operation of addition. Other related terms would include *sum*, *total*, *more than*, and *greater than*.

3. The student will express mathematical terms to describe number relationships.
*Introduce all new terminology while using concrete objects and manipulatives to teach the concept and emphasize its use as you and the student move into working with numbers and number sentences. Make sure that she uses the terminology expressively. For example, you might ask, "How does the number of boys in the class compare with the number of girls?" so that she will answer, "There are more boys than girls." You can then ask her to express the same concept in other ways, such as, "There are fewer girls than boys," and "There are not as many girls as boys."

4. The student will develop and utilize bank of mathematical vocabulary and concepts.
*Have the student develop a written file of math vocabulary. On a 3 x 5 index card, have her write the word, the definition, and make a picture to illustrate meaning. Have the student file the words alphabetically. Provide opportunities for review. Provide the student with clues that will help her to recall specific math terminology. For example, to differentiate between the terms, *numerator* and *denominator*, she can remember that the denominator is down.

BASIC MATH SKILLS:

1. The student will improve computation skills with the use of a number line.
*Teach the student how to use a number line, counting forward and backward, to solve addition and subtraction problems.

2. The student will demonstrate the relationship between basic operations.
*When reviewing the four basic operations, explain and demonstrate to the student the relationship between addition and multiplication and between subtraction and division. For example, demonstrate how 8/2 = 4 means that 2 can be subtracted from 8 exactly 4 times.

3. The student will demonstrate an understanding of the concept of zero.
*Review the concept of zero as used within all the basic mathematical operations. For example, teach the student why one cannot subtract a number from zero (unless using negative numbers), but can add any number to zero. Teach the student why any number multiplied by zero is zero and that zero then becomes the place holder in the answer.

4. The student will demonstrate computation skills horizontally and vertically.
*Teach the student how to interpret problems set up horizontally as well as problems set up vertically.

SELF-MONITORING:

1. The student will develop the self-monitoring strategy of estimation for calculation.
*Help the student develop self-monitoring skills by estimating answers to math calculations. Have her estimate the answer and write it by the side of a problem before she calculates the answer.

2. The student will develop the self-monitoring strategy of self-talk.
*Teach the student to talk through the steps of computation problems as she attempts to solve them.

3. The student will develop the self-monitoring strategy of reverse operation check.
*Teach the student how to check her answers to problems with all types of operations by using the reverse operation. As examples, teach the student to check her division answers by multiplying the divisor by the quotient to get the dividend; for ratio equality by using cross multiplication of terms.

NOTING PROCESS SIGNS:

1. To increase attention of signs, the student will color code math pages prior to completing assignments.
*Before the student works a page of math computation problems, have her color code the operation signs. Help her decide on a color for each of the four signs and consistently highlight or trace each in its own color.

2. To increase attention of process sings, the student will verbalize the needed operation for each problem prior to solving it.
*Teach the student to look at and say the process sign aloud, before she begins to solve a problem.

3. To increase attention of process signs, the student will complete additional mixed process assignments.
*To reduce the student's tendency to overlook operations signs, in all assignments, include a mixture of problems rather than just one operation, such as a page of addition problems.

NUMBER SENSE, NUMERATION, PLACE VALUE

CONCEPT DEVELOPMENT:

1. The student will utilize concrete objects and manipulatives to demonstrate knowledge of concepts.

*Use concrete objects and manipulatives to teach all new concepts and to extend previously presented concepts. Then, if the student is ready, provide similar work with pictures, then tallies, and finally numbers.

2. The student will demonstrate knowledge of mathematical relationships through the use of discovery-oriented activities in addition to the core program.

*Provide the student with discovery-oriented activities that will promote understanding of mathematical relationships including object permanence, conservation of matter, reversibility, seriation, one-to-one correspondence, and classification.

CLASSIFICATION-CREATING SETS:

1. The student will develop skills in classification through sorting activities.

*Introduce the concept of sets and develop classification ability by teaching the student to sort objects, first using familiar objects, then pictures.

2. The student will move from concrete to abstract classification skills through a series of sorting activities.

*To teach the concept of sets and develop classification ability, provide sorting activities such as the following. Using many physical objects, ask the student to sort through them to find all those that match an object shown by the teacher (e.g., all the balls, like this, on the toy shelves). Subsequently, have the student find sets of orally described objects (e.g., all the balls in the room, all the tools that may be used for drawing). Gradually work with more abstract objects and attributes of objects such as pictures, then shapes.

3. The student will develop the concept of classification through identification of similarities and differences.

*Use activities to develop the concept of classification that are based on finding similarities and differences between and among objects. Examples include: 1) Using a variety of objects, such as small, toy chairs, cars, people, and food, place one of each kind in each of two or more shoeboxes. Have the student sort through the pile of objects provided to find similar items and sort them into the appropriate boxes. Do the same activity using pictures cut from magazines. 2) Using manipulatives such as Attribute Logic Blocks, Cuisenaire Rods, and flannel boards, guide the student to discover a variety of attributes by which these materials may be sorted. Attributes include color, shape, size, thickness, width, height, and use. 3) To provide practice in classifying and creating sets, create experiments in which the students participate. Examples of sets amenable to experimentation are objects that float/sink, those that feel heavy/light, and those that can be seen through/those that cannot.

4. The student will develop flexibility of classification through guided exploration.
*If the student is having difficulty with number concepts because she has not yet developed flexibility in working with classification and multiple attributes of sets (groups). Provide guided exploration in classification activities and once she understands the concept of grouping according to similarities and differences, extend the concept to flexibility of classification (i.e., grouping objects/pictures based on similarity of attribute, then regrouping the same objects/pictures based on a different attribute). Additional suggestions are: 1) Take a set of objects and divide them into groups based on a particular attribute (tools and cooking utensils). Have the student explain on what basis you separated them (What are the differences between groups and what are the similarities within groups?). 2) Reassemble the objects into one group and ask the student to group them on some other basis (e.g., metal and wood) and discuss the attribute on which she made the distinction (in this case, what they are made of). 3) Continue taking turns classifying objects in different ways. 4) When the student appears to have a solid understanding of this concept, extend the concept by using two attributes to differentiate between groups (e.g., height and function). 5) Progress to more complex relationships (e.g., color or function). 6) Gradually move to more abstract materials such as Attribute Logic Blocks, Tangrams, and Cuisenaire Rods, and eventually to number and letter patterns.

FROM SETS TO NUMBERS:

1. The student will develop the concept of what makes sets through guided activities.
*Teach the student that the members of a set may or may not contain objects/pictures that have a common attribute. For example, a star, a ball, and a tree may all share membership in a set just because they are described as such.

2. The student will develop the concept of the empty set through guided activities.
*Provide activities to develop the concept of the empty set, the idea that a set may have no members. For example, give four students boxes with three, two, one, and no crayons in them, respectively. Have each student describe the contents of their box, ending with the student who has no crayons. Explain that her box contains the empty set. Or, ask questions for which the answer is "none," such as the number of members in the set of zebras in the classroom.

3. The student will develop the concept of naming sets by attributes (classification) through guided activities.
*Teach the student that sets in which members have a common attribute may be named for that attribute. For example, a group of colored objects may be sorted into a red and a green set.

4. The student will develop the concept of representing sets by the amount of items (number) through guided activities.
*Help the student to develop conceptualization of sets at the semi-concrete level by representing groups of physical objects by pictures. When the student clearly understands the representation, introduce new concepts with pictures. Assess comprehension by asking questions about the relationship among objects in the environment.

5. The student will develop conceptualization of sets at the semi-abstract level through guided activities.
*Help the student to develop conceptualization of sets at the semi-abstract level by teaching her to represent each member of a set with a tally, such as a Popsicle stick. This will encourage her to consider the number properties of sets rather than other characteristics of the objects.

COUNTING AND ONE-TO-ONE CORRESPONDENCE:

1. The student will demonstrate automaticity of one-to-one correspondence through systematic practice.
*When the student is counting small items, teach her to separate each from the group as she counts it so that she does not miss any or count the same item twice. Provide systematic practice until the procedure is automatic.

2. The student will develop the concept of one-to-one correspondence through guided practice.
*Engage the student in activities to develop the concept of one-to-one correspondence. This is a prerequisite to understanding addition and subtraction.

3. The student will demonstrate one-to-one correspondence to compare quantities and use associated vocabulary.
*Teach the student to use one-to-one correspondence to compare quantities; teach the associated vocabulary. For example, have her match each member of one set with a member in another set. Help her verbalize if one set has the same number as the other set or not. Gradually, introduce comparative terms such as more, less, same, equal and have her use these terms in statements describing the relationship between the sets (e.g., "There are more shoes than stars;" "There are the same number of shoes as stars").

4. The student will develop conservation of numbers through guided practice.
*The student does not count on because she is still unsure that the quantity of a group of objects is invariable (if nothing has been done to it). Students who count on assume that 6 is really 6 so they do not have to count it. To help the student understand the process of counting on, have her count all the objects in two sets and write the answer. Then guide her in counting on and in comparing the answers. She may need to practice this many times to understand the validity and efficiency of the latter approach.

5. The student will develop conservation of numbers by comparing sets.
*While working on comparing the number of items in two sets, work on developing the concept of conservation of matter. Help the student to see that one has the same number of objects, no matter how they are arranged, the same number of objects is still present. Engage the student in activities to experience conservation of other attributes, such as volume and length.

USING NUMBERS:

1. The student will develop symbolic representations through review of readiness skills.
*Do not introduce numerals to the student until she is developmentally ready to use the symbolic representations. Make sure that she understands one-to-one correspondence by providing appropriate readiness experiences, such as counting objects and matching or comparing one set of objects to another.

2. The student will associate number symbols to items through guided activities.
*Use a variety of activities to guide the student to associate number symbols with the amount of items within a set, such as matching large models of numbers (e.g., plastic, rubber) with cards according to the number of pictures on each.

3. The student will develop rapid recognition and conservation of numbers through instructional games.
*Use games to reinforce the idea of numbers representing a set and rapid recognition of the number of items in a set. For example, play a version of Bingo in which the teacher holds up a card with a set of dots on it and the students cover the corresponding number on their Bingo cards. To reinforce rapid recognition of item amounts up to [3, 5], the teacher can show the cards for decreasing intervals of time.

4. The student will develop conceptual basis for addition through guided practice with sets.
*As a conceptual basis for addition, using manipulatives or pictures, teach the student how to join two sets so that all members of the first set and all members of the second set are combined to form a new set. When this concept is mastered, have the students give each of the sets a number (in writing or with number tiles).

5. The student will manipulate objects to demonstrate knowledge of sets and subsets in guided activities.
*Use objects or pictures to teach the student that sets may be broken up into a variety of subsets. For example, a set of five objects contains: five subsets of one member each; one subset of two and one subset of three; one subset of four and one subset of one; or one subset of five and one subset of zero.

NUMBER PATTERNS AND ORDINAL NUMBERS:

1. The student will develop the concept of pattern recognition through guided activities.

*Help the student develop the concept of pattern recognition using familiar objects in simple patterns, such as fork, spoon, fork, spoon, fork, spoon. Guide the student to describe the pattern. Very gradually, increase the complexity of the pattern and the abstractness of the items in the pattern (e.g., from objects to colored beads to pictures to amount of items in pictures). Eventually, move to recognition of simple number patterns. At each level, when the student is able to identify the pattern, have her complete similar patterns.

2. The student will use a number line to develop concept of number sequencing.

*Help the student develop the idea of number sequence. Use a number line to answer questions such as: "What number comes just before...?" "What number comes just after...?" Work with segments of numbers and fade use of the number line as each segment is mastered. Provide a wide range of activities so that the student learns that *before*, *after*, and *between* relate to the sequence of numbers in general (abstract) not just on the number line.

3. The student will recognize ordinal and cardinal numbers through gross-motor activities.

*Reinforce the use of ordinal numbers with gross-motor activities such as having a few students stand on a number line on the floor. After each one says the cardinal number, tell the student the corresponding ordinal number (e.g., "Joel is first in line, Desiree is second"). Then, using ordinal numbers instead of names, give each student an instruction (e.g., "Will the second child clap her hands two times?").

4. The student will recognize numbers by using strategy of color-coding of repeating patterns.

*As the student learns to recognize numbers, color-code the repeating patterns of the numerals in the units column, then the tens columns, and the hundreds column.

RECOGNIZING AND WRITING NUMBERS:

1. The student will form a stable visual image of numbers through multisensory activities.

*Provide the student with many opportunities to form a stable visual image of numbers before she is asked to write them. Suggestions include playing with number puzzles; placing number models in sequence; matching number tiles, cards, or models; and feeling a number model with eyes closed and naming it.

2. The student will master number recognition and counting skills through guided practice of games/activities.
*At home or at school, use games to reinforce number recognition and counting skills. For example, play board games that require moving a marker a specified number of squares, card games such as 21," or dominoes with a peer or parent who will encourage accurate counting with comments such as, "See if you have a domino with three dots just like this one."

3. The student will recognize cardinal numbers by name through gross-motor activities.
*Teach the student to recognize the cardinal numbers by their names. You may use gross-motor activities to reinforce number recognition by writing the numbers to 10 on a long sheet of butcher paper and having the child count her paces, simultaneously stepping on each square and looking at the number. Alternatively, you may use numbers on steps.

4. The student will develop readiness skills of writing numbers through multisensory experiences.
*To help the student get ready to start writing numbers, provide multisensory experiences to help familiarize the student with the form. For example, the student may copy a model to make numbers out of clay, make cookies in the shape of numbers, trace numbers in wet sand or on tactile number cards.

PLACE VALUE:

Ensure that the student has developed a solid understanding of place value before introducing regrouping (borrowing and carrying).

1. The student will visualize place value through the use of stories.
*When teaching place value, tell the student a story with pictures to help her understand that no more than 9 of any set may go in one column. The following is an example of a place value story with related pictures.

> Once there was a person who moved into a house. Although the landlord had told her that no more than 9 people could live in the house, eventually a tenth person moved in. The landlord said, "Only 9 people can live in that house," and he evicted all of them. So, they became one family (of 10 people) and moved into an apartment house. The apartment house had room for 9 families just like theirs—10 people each. But families kept moving into the apartment house and before they knew it, there were 9 families there. Eventually, a tenth family (of 10) moved in. When the landlord found out, he said, "Only 9 families can live in that house." And he threw them all out. So the 10 families formed a community (How many families are in that community now? How many people?). The community then moved into a huge, brand-new apartment complex. This new landlord told them, "Only 9 communities can live in this complex." And they said, "OK," but, eventually, other communities (of 100) moved in. Finally, when the tenth community moved in.

2. The student will develop the concept of place value through guided activities using manipulatives.
*Using manipulatives (e.g., Unifix cubes, Base Ten Blocks), teach the student the concept of place value. Teach her how to "trade up" (regroup) for the next largest set and how to use a place value mat. If you use a dry erase board for the mat, she can write the digits corresponding to the blocks in each column. Write the resulting number on another piece of paper. Try to guide her to discover and verbalize that the value of the digit within the number is related to its position on the board and the way it is represented by the objects (e.g., small blocks, sticks equivalent to ten blocks, flats equivalent to 100 blocks or 10 sticks).

3. The student will demonstrate knowledge of place value and printed numbers through guided practice.
*When the student has learned the concept of place value, show her how reading printed numbers relates to the place value of the digits. Write the number, have the student place the corresponding number of blocks on the mat, and have her read it. Point out that the digit with the highest value is named first. When she understands this, reverse the process so she learns to write multi-digit numbers.

4. The student will work with expanded notation by using place value knowledge.
*When the student understands the connection between the place of the digits comprising a number and the value of the digits, extend the concept to working with expanded notation.

5. The student will demonstrate the function of zero in place value by using manipulatives an organized structure.
*Review or reteach place value using manipulatives to clarify the function of zero as a place holder and as representing an empty set. Use a place value mat or a box divided into parallel compartments. Represent multiple-digit numbers on the mat/box and practice reading them. Start with no zeros in the number, then one zero, then two. Transfer the activity to reading numbers written on paper.

6. The student will verbalize place value concept through explicit connection of paper work and manipulatives.
*Explicitly connect regrouping on the place value board with regrouping using numbers on paper. Have the student work the printed problems by doing them on paper and the board. When she moves a ten-stick (or any manipulative representing a group of ten) over into the tens column, she puts the one in the box above the tens column in the printed problem. Give the student lots of practice in verbalizing what she is doing as she works the problems both on the board and on the paper.

BASIC FACTS

1. The student will develop automaticity of math facts through strategy of overlearning.
*Have the student overlearn the math facts for all operations. Make adequate provisions for overlearning by using games, language masters, tape recorders, computer software, tracing activities, and peer tutoring.

2. The student will utilize a multiplication chart to develop automaticity of math facts.
*If the student has historically been unable to recall math facts, develop strategies to solve computation problems. For example, give her pocket-sized charts with addition and multiplication facts. Teach her how to use the addition chart for subtraction and the multiplication chart for division. If she has difficulty finding the number at the intersection of the row and column for the fact she is checking, provide a template made from strips of colored transparency material in the shape of a reversed L. At the juncture of the two strips, cut out a square through which the answer can be seen. The student may use the chart any time she wants for the answers to math facts but encourage her to guess the answer before looking. That way, she gets immediate reinforcement if she is right and immediate correction if she is wrong. Additionally, eventually, visualizing the location of the answer may help in retrieving it. When a fact has become truly automatic, she should black it out. This will continuously strengthen her ability to recall the fact as well as allow her to see how many facts that she has left to learn

3. The student will use strategic organization to memorize math facts.
*The student creates elaborate strategies for reconstructing the answers to simple math fact problems because she has not memorized basic math facts. As she has demonstrated understanding of the concepts underlying [addition, subtraction, multiplication, division], organize the facts for easiest learning and recall, then provide daily drill and practice. Use a systematic plan for practice so that previously learned facts are reinforced along with more recently learned ones.

2. The student will use skip counting and hatch marks to solve computation problems.
*To reduce memory demands, teach the student strategies to use to solve computation problems based on facts she already knows. Use skip counting and hatch marks (e.g., for 5x4, count by fives, stopping when you have made 4 hatch marks)

3. The student will use repeated addition to solve computation problems involving multiplication.
*Use repeated addition for the lower multiplication tables

4. The student will use a split factor and add strategy to solve computation problems.

* Teach the student to split one factor in half and add the products (e.g., 4 x 7 → 2 x 7 = 14, and 14 + 14 = 28, so 4x7=28);

5. The student will develop flexibility and understanding of number relationships through number sentence strategy.

*While teaching addition/ multiplication facts, help the student to develop flexibility and a deeper understanding of number relationships by working with addition/ multiplication sentences (e.g., 2+5=7). This will reinforce the addition/multiplication facts while increasing readiness to learn the subtraction,/division facts. Provide a variety of guided activities with addition /multiplication sentences in which one of the numbers or relational signs is omitted. Have the student figure out the number/sign that would make the sentence true and explain how she did so. Use the fact groups the student is studying for the arithmetic sentences. As she develops facility in filling in the missing element, she will be able to come up with the related[subtraction,/division facts. The ability to complete 6+2=8=6+ﻕ ,8=ﻕ+6 ,ﻕ, and 28=6ﻕ, facilitates comprehension and acquisition of 8–2=6 and 8–6=2.

6. The student will learn addition facts through family structure strategy.

*Try teaching addition facts as "families" centered on sums. For example, the "3 family" has 4 "brother-sister pairs:" 0+3, 1 + 2, 2 + 1, 3 + 0. Given three blocks and 2 empty frames, the student can practice making different brother-sister pairs and matching them to number cards (as above) and then to addition fact cards. This type of structure may be easier for her to learn than the usual +1, +2, +3. One advantage to teaching facts in this way is that in any subtraction fact, the higher number is the family name and the other number is one of the brother-sister pairs. The answer is the missing brother-sister pair.

7. The student will develop automaticity of multiplication facts by using the write-say strategy.

*Use a write-say procedure to help the student memorize her multiplication facts. Give the student a worksheet of problems with answers. Have her cover the row of answers with an index card. Have her look at the problem, say the answer, and then move the card to check the answer. If the answer is incorrect, have the student look at the problem and answer, cover it with a finger, and then write it from memory several times, checking for accuracy each time. Assign only a few facts at a time.

8. The student will develop automaticity of math facts through audio-visual strategy.

*Record a set of number facts on tape at a slow pace, repeating each one twice. Have the student hold a flash card in her hand as she listens to each fact being said. Encourage the student to repeat the fact aloud with the taped presentation. As an alternate activity, have the student say, then write, each math fact. Present only a few at a time. Provide daily practice for 5 minutes until all facts are mastered.

9. The student will develop automaticity of math facts through visual stimuli.
*To help the student increase her speed in math operations, drill her on math facts using visual stimuli such as flashcards, computer programs, and, when she can respond to a math fact within 3 seconds, worksheets. Eventually, move to timed tests.

10. The student will develop automaticity of math facts through Precision Teaching technique.
*Use Precision Teaching to help the student memorize her math facts. Have her complete daily timed drill activities where she competes against her own best score

PROGRAMS

- *Touch Math*: a systematic instructional program for teaching computation skills. Touch Math uses dots on each number to represent its value, emphasizes use of the Touchpoints as a bridge to conceptualizing and memorizing the facts, and provides visual prompts to help students learn the algorithms. Particularly helpful for students who experience great difficulty memorizing and retrieving math facts

- *Great Leaps Math Program* (Mercer, Mercer, & Campbell, 2002): helps to build fluency in the basic facts, including addition, subtraction, multiplication, and division

ALGORITHMS

- Reteach the concept of fractions, including what the numerator and denominator represent, the reason for needing a common denominator, the methods for finding common denominators, the procedures for reducing fractions, and the meaning of improper fractions and mixed numbers.

- Before introducing decimals, make sure that the student understands fractional concepts so that she can see the relationship between the two and acquire an understanding that decimals are an easier and more consistent way to express fractions.

- To provide a basis for higher-level math, help the student develop concepts and skills with percents, decimals, fractions, and negative numbers.

ALGORITHM SEQUENCE INTERVENTIONS:

1. The student will use a flow chart to _____.
*Make flowcharts for the student to illustrate the sequence of steps required for any particular operation that she is learning. Have the student keep the flowcharts clipped inside her math textbook and/or workbook to refer to whenever necessary.

2. The student will use a cue card to _____.
*Provide the student with an index card that contains clear verbal explanations of questions to ask herself as she works math problems. For example, when learning regrouping techniques for subtraction, write the question "Is the top number larger than the bottom number?"

a. If yes – subtract If no - regroup (borrow)

3. The teacher will conduct an error analysis and provide provide immediate feedback through the development of decision map for _____.
*Analysis of the student's errors indicate that she understands that the procedures are rule-governed but that she misunderstands or misapplies them, or has made up a rule for herself. To help her use and remember the correct rules to use, provide her with memory aids such as decision map.

4. The student will correctly sequence _____ with the use of memory strategies.
*Teach the student memory strategies for performing new math algorithms in the correct sequence. For example, for long division, teach her to write the symbols representing the steps at the top of her paper, to recite, "Divide, multiply, subtract, check, bring down," or make a tune for it and sing it

5. The student will use taped instructions of computation steps to solve problems.

*Provide tape-recorded instructions for solving computation processes. Fade use of the tape gradually as the student memorizes the steps.

6. The student will use color coding strategy to _____.
*Use color coding to: (a) identify starting and stopping places within a problem; (b) code the units, tens, hundreds, and thousands place; (c) indicate where the final answer should be written; and (d) highlight important features, such as operation signs, the question being asked, or the key information being asked in the problem.

7. The student will use a visual cue as a reminder for computation start points.
*Remind the student that reading starts on the left and moves right, whereas the math computations of addition, subtraction, and multiplication start on the right and move left. If she requires a visual cue to remind her, place a green dot or arrow over the units column and have her place her pencil point on the dot or arrow before beginning the problem.

MEASUREMENT AND ESTIMATION

1. The student will use visual or auditory associations to remember units of measurement.
*When the student has to memorize rote information, such as units of measurement, help her to create visual or auditory associations to help her remember them. For example, 12 inches in a foot could be drawn as a footprint of a bare foot with 12 inchworms end to end crawling across it. To add in that there are 3 feet in a yard, 2 more feet could be drawn, toe to heel with the first foot (without the worms) tightly bordered by a fence filled in with grass to represent a "yard."

2. The student will differentiate between exact or estimate answers are needed through guided practice.
*Help the student learn to differentiate situations where an exact answer is needed from those where an estimate is more appropriate.

3. The student will use non-standard units of measurement to estimate length.
*Teach the student to estimate length in terms of familiar objects. For example, a regulation baseball bat is 1 meter in length. A notebook is approximately a foot in height.

APPLICATION

PROBLEM-SOLVING:

1. The student will substitute smaller numbers to work through operations, then refigure using original numbers.
*When the student has difficulty with the computation involved in a story problem, have her substitute smaller numbers so that she can understand the operation(s) involved and then calculate the problem a second time using the original numbers.

2. The student will develop a variety of strategies from which to draw from when solving math problems.
*Teach the student how to plan what she needs to do to solve a problem. Different techniques, such as the following, may be called for by the type of question asked: 1).*Decide what operation*(s) to use (e.g., Harry weighed 250 pounds. He weighed 72 more pounds than James. How much did James weigh?) 2) *Make a table, graph, or chart* of the information provided (e.g., Hansel and Gretel went to the witch's house every day except Sunday. On Mondays and Thursdays, Hansel went twice. On Wednesday, Gretel went in the morning, at noon, and once after Hansel was in bed. Who traveled to the witch's house more times in a month?) 3) *Make a drawing of the information* provided (e.g., Mehitabel planted a square garden with 12 garlic plants on each side to keep the snails away. How many garlic plants did she plant?) 4) *Make inferences and logical deductions* (e.g., The Carsons went to Jack-in-the-Bag and spent $20.75 for lunch. An adult meal costs $4.95 and lunch for a child costs $2.95. How many people are in the family? How many of them are children?)

*Other strategies to teach students: restating the problem; acting out the problem; estimating the answer; using a model; guessing and checking; making a drawing; working backward; solving a simpler, related problem; constructing a table or graph; looking for a pattern; applying a formula; and writing a mathematical sentence.

3. The student will use a cue card with question strategy to solve story problems.
*Teach the student a simple question strategy to use for solving story problems. For example, teach her to: (a) read the problem, (b) reread the problem to identify what is given (What do I know?) and to decide what is asked for (What do I need to find out?), (c) use objects to solve the problem and identify the operation to use, (d) write the problem, and (e) work the problem (Smith & Alley, 1981). Write the strategy on an index card for easy reference.

HIGHER LEVEL MATHEMATICS

PROBLEM SOLVING

1. The student will work at a modified pace with spiral review of previously covered material.

*If the student has not automatized many procedures commonly applied in algebraic problems, the cognitive attention necessary to reason through the problem is diverted to the steps in the procedures. To overcome this problem, she requires far more practice with a variety of problems addressing each new concept and procedure taught, and extensions of those previously taught. This will necessitate moving through the curriculum at a slower pace and periodically reviewing material covered previously.

2. The student will convert relational statements to mathematical expressions through interpretation and paraphrasing techniques.

*If the student appears to have difficulty converting relational statements (e.g., a plane travels 10 times faster than a car, Tia's grade is 17 points higher than Robin's) into mathematical expressions, focus instructional time on interpreting and paraphrasing a wide variety of relational statements and then representing them in mathematical form.

3. The student will recognize mathematical structures through comparison of problem structures.

*To help the student recognize the mathematical structure of word problems, provide direct instruction in comparing a group of word problems and identifying those with the same mathematical structure by drawing representations of the problem structures (i.e., equation network), for example, with a chart or diagram (d'Ailly, 1995).

4. The student will use models to assist in recognizing problem structures.

*When presenting a new type of problem or equation, in addition to instructing the student in how to work them, give her examples that have already been worked. Give her time to study them, answer any questions she has, and have her use them as models while working problems with the same structure. This strategy should help her recognize the problem structure at a later time and facilitate the solution.

5. The student will comprehend word problems and level of abstraction through systematic instruction (concrete factual, concrete hypothetical, abstract factual, abstract hypothetical).

*The student's difficulty with comprehending word problems increases along with level of abstraction. She will have more success if, within each procedure or concept presented, concrete factual problem types are introduced first, concrete hypothetical second, abstract factual third, and abstract hypothetical, the most difficult, last. Examples of each problem type follow: 1) *Concrete Factual*: A farmer has eight more hens than dogs. Since hens have two legs each, and dogs have four legs each, all together the animals have 118 legs. How many dogs does the farmer own? 2) *Concrete Hypothetical*: There are four more girls in an English class than boys. If there were six times as many girls and twice as many boys, there would be 136 pupils. How many boys are there? 3) *Abstract Factual*: The value of a given number is six more than the value of a second number. The sum of two times the first number and four times the second number is 126. what is the value of the second number? 4) *Abstract Hypothetical*: A given number is six more than a second number. If the first number were four times as large and the second two times as large, their sum should be 126. what is the second number? (Caldwell & Goldin, 1987).

6. The student will utilize a situational model as a intermediate step to quantitative representation.

*To help the student learn to analyze and then reintegrate the information in a problem, teach her to create a situational model (e.g., picture, diagram) of the problem before trying to set up a quantitative representation such as an equation. Research indicates that inclusion of this intermediate step (the situational model) is positively related to correct solutions (d'Ailly, 1995).

7. The student will construct similar problem with fewer variables to clarify understanding, then adapt solution to original problem.

*If the student is confused by problems that contain many variables, the following strategy may be helpful. Teach her to construct a different problem, similar to the one that is given but with fewer variables and solve it. This may help her to clarify her understanding of the given problem using all of the variables and allow her to solve it by adapting the solution she used in the simpler problem (d'Ailly, 1995).

8. The student will break larger tasks into smaller more manageable parts, then combine results.

*Teach the student that when a problem appears too confusing to start on, to try and find part of the answer and see if she can proceed from there. Alternatively, she can try to break the problem into smaller or simpler questions, solve them, and combine the results (d'Ailly, 1995).

9. The student will organize algebraic word problems in a 6-step procedure.

*To aid the student in organizing her approach to algebraic word problems, teach her to follow the following 6-step procedure: (1) Read the problem carefully; (2) Decide what question the problem asks and choose a variable to represent the unknown; (3) Consider the other information given in the problem and how it relates to the unknown; (4) Write an equation or equations expressing the given relationships; (5) Solve the equation or equations; (6) Check the answer. (Bassler et. al, 1975).

10. The student will use a Contrapositive strategy to prove a logic statement.
*Contrapositive: When the statement that must be proven is positive (e.g., If X is true, then Y must be true), instead try to prove the equivalent negative statement (e.g., If Y is false then X must be false). Either is a sufficient proof.

11. The student will use a Contradiction strategy to prove a logic statement.
*Contradiction: Assume that the statement that must be proven is false. Using this assumption, the student tries to prove either that one of the conditions given in the problem is false, that something she knows to be true is false, or that what she wishes to prove is true. If she can do any of these, she has proved the original statement. (Schoenfeld, 1979).

12. The student will answer guided questions to develop awareness of problem-solving strategies.
*Guide the student learn to think about how she tried to solve the problems and to comment on what she did well and how she could improve her performance. Ask questions such as:

- How did you do this problem?
- I noticed you had some trouble with _____. How did you proceed?
- How could you solve the problem in another way?

13. The student will use self-questioning strategies to represent algebra word problems.
*Use the following types of self-questioning strategies to help the student learn how to represent algebra word problems (Hutchinson, 1993):

- Have I read and understood each sentence? Are there any words whose meaning I need to ask?
- Have I gotten the whole picture of the problem?
- Have I written down my representation of the problem on my worksheet? (goals; unknown(s); known(s); type of problem; equation)
- What should I look for in a new problem to see if it is the same kind of problem?

14. The student will use self-questioning strategies to represent algebra number problems.

*For solving algebra number problems:

- Have I written an equation?
- Have I expanded the terms?
- Have I written out the steps of my solution on the worksheet? (collected like terms; isolated unknown(s); solved for unknown(s); checked my answer with the goal; highlighted my answer)
- What should I look for in a new problem to see if it is the same kind of problem?

PROGRAMS:

- *Today's Mathematics*: 10th Edition: Part 1: Concepts and Classroom Methods (Heddens & Speer, 2000a), and Part 2: Activities and Instructional Ideas (Heddens & Speer, 2000b). These companion books provide comprehensive and detailed information regarding national math standards, a graded scope and sequence for curriculum expectations, developmental levels within strands of mathematics, explanation of and instructional approaches in developing mathematical concepts and procedures from readiness through beginning high school math, and integration of technology, as well as specific activities and suggestions for classroom instruction.

- *Teaching Mathematics to Students With Learning Disabilities, Fourth Edition* by Carol A. Thornton and Nancy S. Bley (Pro-Ed, 2000)

- *Windows of Opportunity: Mathematics for Students With Special Needs* edited by Carol A. Thornton and Nancy S. Bley (National Council of Teachers of Mathematics, 1994). For specific suggestions for teaching mathematics to students with special needs

- The following *websites* are excellent for checking out the Principles and Standards for School Mathematics for instructional principles, ideas, and examples [National Council of Teachers of Mathematics, 2000]: http://www.nctm.org and http://www.illuminations.nctm.org.
- AskERIC. The website is http://ericir.syr.edu/.

- *Optical Data School Media*, a subsidiary of SRA/McGraw-Hill: a source of videodisks that provide visual representations of math concepts and practice in problem-solving, and that may be used to supplement the existing math program.

- *Touch Math:* For the student who is having difficulty memorizing math facts and the steps of the basic operations, supplement her current math program with a multisensory program, such as Touch Math.

- *Touch Math Story Problems Kit:* This program provides for reinforcement of the concepts of each operation, instruction in the related language, and application to practical problems. This is a good resource for the student that needs a highly structured, multisensory, incremental approach to learning the language of math for comprehension of word problems.